Written Exam at the Department of Economics winter 2017-18

# Microeconomics III

Final Exam

Date: January 6, 2018

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

**This exam question consists of 4 pages in total**

*NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.*

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. Consider the game below in Figure 1. The first payoff is that of player 1, the second that of player 2.

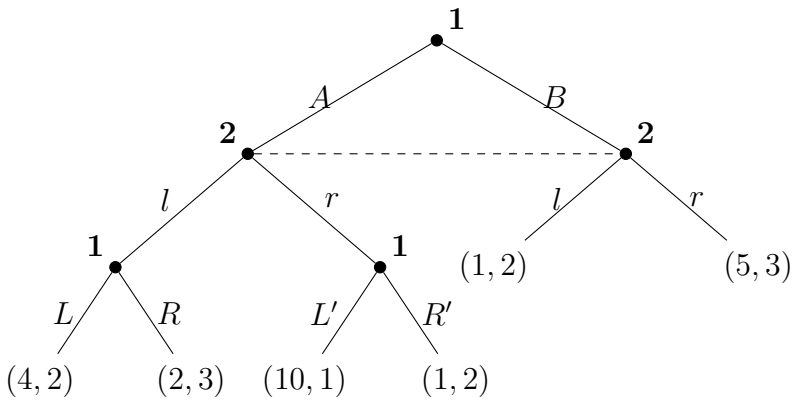


FIGURE 1: DYNAMIC 2-PLAYER GAME

(a) Is this a game of perfect or imperfect information? How many strategies does each player have? How many proper subgames are there (not including the game itself)?

(b) Find the set of pure-strategy subgame-perfect Nash equilibria (SPNE) of the game.

(c) Now suppose that player 2 observes player 1's choice of $A$ or $B$. Draw the game tree of the modified game, and find the set of pure-strategy SPNE.

(d) Compare the outcome and payoffs of the SPNE you found in the original game with the outcome and payoffs of the SPNE in the modified game. If they are the same, comment on why this is. If they are different, explain what causes the difference.

(e) Can you change one payoff (but just one) of one of the players such that there then exist an SPNE of the original game and an SPNE of the modified game that yield the same equilibrium payoffs? If yes, indicate which payoff can be changed and show the solution. If no, argue why this is.

2. Consider the signaling game shown in Figure 2 below.

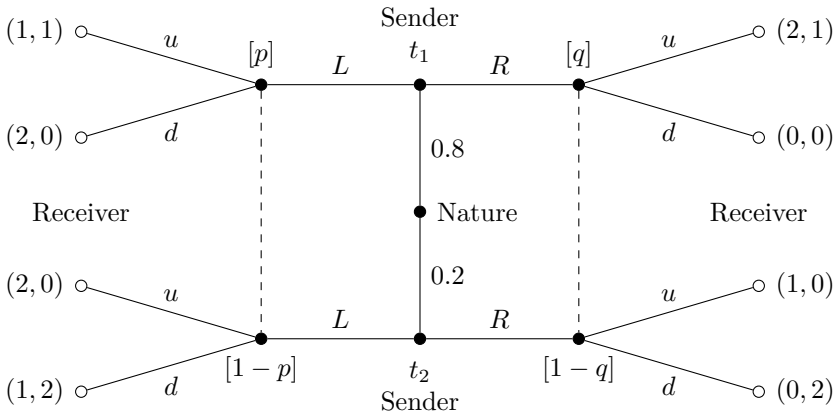


FIGURE 2: SIGNALING GAME

(a) Show that there exists a pooling perfect Bayesian equilibrium (PBE) in which both Sender types play $L$. Be careful to specify the beliefs $p$ and $q$ that support this equilibrium.

(b) Does this pooling PBE satisfy SR5 and SR6?

(c) Are there any separating PBE? If yes, show that one such equilibrium exists. If no, demonstrate that no such equilibrium exists.

3. Consider the following version of Spence's education signaling model, where a firm is hiring a worker. Workers are characterized by their type $\eta$, which measures their ability. There are two worker types: $\eta \in \{L, H\}$. Nature chooses the worker's type, with $p = \mathbb{P}(H)$. The worker observes his own type, but the firm does not.

   The productivity $y$ of the worker depends only on his type: $y(\eta, e) = \theta_\eta$. Education is thus non-productive. Assume that $\theta_H = 2$ and $\theta_L = 1$.

   The worker can choose his level of education: $e \in \mathbb{R}^+$. The cost to him of acquiring education is

$$c(e, \eta) = \frac{\sqrt{e}}{\theta_\eta}.$$

   Education is observed by the firm, who then forms beliefs about the worker's type: $\mu(e) = \mathbb{P}(H|e)$. We assume that the firm is in competition such that it pays the expected productivity:

$$w(e) = \mathbb{E}(\theta_\eta|e),$$

   where the expectation is calculated given the firm's beliefs $\mu$ about the worker's type.

   The payoff to a worker conditional on his wage, type and education is

$$u(w, \eta, e) = w - c(e, \eta).$$

   We will look for pure-strategy perfect Bayesian equilibria (PBE). Denote the equilibrium level of education chosen by the two types, respectively, by $e_H^*$ and $e_L^*$.

   (a) First, we will look for a separating equilibrium in which $e_H^* = 1$ and $e_L^* = 0$. Throughout this part, you can assume that the off-equilibrium-path beliefs are $\mu^*(e) = 0$ if $e \neq e_L^*, e_H^*$.

      (i). Specify the beliefs that must apply on the equilibrium path.

      (ii). Then argue that given the beliefs, the worker should only ever choose either $e = 0$ or $e = 1$.

   (b) Using your answer to the previous question, show that there is a separating PBE where $e_H^* = 1$ and $e_L^* = 0$. Be sure to fully specify beliefs and equilibrium strategies.

   (c) Continue to assume that the off-equilibrium-path beliefs are $\mu^*(e) = 0$ if $e \neq e_L^*, e_H^*$. Also, continue to consider $e_L^* = 0$. Find all the values of $e_H^*$ such that a separating PBE exists. Be sure to fully specify beliefs and equilibrium strategies.

   (d) Now apply SR6 (equilibrium dominance). Which of the equilibria you found in (c) satisfy SR6?

   (e) Comment on SR6. Do you think it is a reasonable requirement? Explain your answer.